**United Kingdom
Mathematics Trust**

Intermediate Mathematical Challenge
Thursday 6 February 2020
© 2020 UK Mathematics Trust

supported by [XTX] MARKETS Overleaf

> *England & Wales: Year 11 or below*
> *Scotland: S4 or below*
> *Northern Ireland: Year 12 or below*

Instructions

1. Do not open the paper until the invigilator tells you to do so.

2. Time allowed: **60 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.

3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.

4. **Use a B or an HB non-propelling pencil.** Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.

5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.

6. **Scoring rules:**
   5 marks are awarded for each correct answer to Questions 1-15;
   6 marks are awarded for each correct answer to Questions 16-25;
   Each incorrect answer to Questions 16-20 loses 1 mark;
   Each incorrect answer to Questions 21-25 loses 2 marks.

7. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.

8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

1. What is the value of $2 - (3 - 4) - (5 - 6 - 7)$?

   A 11   B 9   C 5   D −5   E −7

2. Which one of these is a multiple of 24?

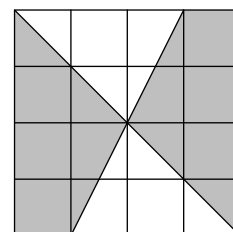   A 200   B 300   C 400   D 500   E 600

3. What is the difference between 25% of £37 and 25% of £17?

   A £4.25   B £5   C £6   D £7.50   E £9.25

4. What fraction of this diagram is shaded?

   A $\dfrac{13}{32}$ B $\dfrac{1}{2}$ C $\dfrac{9}{16}$ D $\dfrac{5}{8}$ E $\dfrac{13}{16}$

5. Four of the following coordinate pairs are the corners of a square.
   Which is the odd one out?

   A (4, 1)   B (2, 4)   C (5, 6)   D (3, 5)   E (7, 3)

6. Which of the following has the largest value?

   A 2^6 B 3^5 C 4^4 D 5^3 E 6^2

7. Kartik wants to shade three of the squares in this grid blue and Lucy wants
   to shade the remaining two squares red. There are ten possible finished
   grids.
   In how many of the finished grids are Lucy's red squares next to each
   other?

   A 3   B 4   C 5   D 6   E 8

8. One of these options gives the value of $17^2 + 19^2 + 23^2 + 29^2$. Which is it?

   A 2004   B 2008   C 2012   D 2016   E 2020

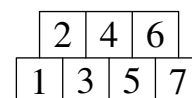9. Adam's house number is in exactly one of the following ranges. Which one?

   A 123 to 213 B 132 to 231 C 123 to 312 D 231 to 312 E 312 to 321

10. What is the value of $\dfrac{2468 \times 2468}{2468 + 2468}$?

    A 2   B 1234   C 2468   D 4936   E 6 091 024

11. I start at square "1", and have to finish at square "7", moving at each step to a
    higher numbered adjacent square.
    How many possible routes are there?

    A 7   B 9   C 10   D 11   E 13

12. Farmer Fatima rears chickens and goats. Today she returned from market and said, "I sold 80 animals, and now there are 200 fewer legs on my farm than before!"
How many goats did she sell?

    A 15   B 20   C 25   D 30   E 35

13. What is half of 1.6×10^6?
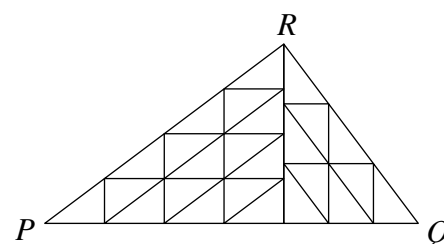
    A 8×5^6 B 4×10^6 C 8×10^5 D 8×10^2 E 1.6×10^3

14. The result of the calculation $9 \times 11 \times 13 \times 15 \times 17$ is the six-digit number '$3n8185$'.
What is the value of n?

    A 2   B 4   C 6   D 8   E 0

15. Triangle PQR has been divided into twenty-five congruent right-angled triangles, as shown. The length of RP is 2.4 cm.

What is the length of PQ?

    A 3 cm   B 3.2 cm   C 3.6 cm   D 4 cm
    E 4.8 cm

16. As a decimal, what is the value of $\dfrac{1}{9} + \dfrac{1}{11}$?

    A 0.10   B 0.20   C 0.2020   D 0.202 020   E $0.\dot{2}\dot{0}$

17. The Knave of Hearts stole some tarts. He ate half of them, and half a tart more. The Knave of Diamonds ate half of what was left, and half a tart more. Then the Knave of Clubs ate half of what remained, and half a tart more. This left just one tart for the Knave of Spades.
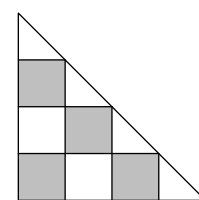How many tarts did the Knave of Hearts steal?

    A 63   B 31   C 19   D 17   E 15

18. The diagram shows an isosceles right-angled triangle which has a hypotenuse of length y. The interior of the triangle is split up into identical squares and congruent isosceles right-angled triangles.
What is the total shaded area inside the triangle?

    A $\dfrac{y^2}{2}$ B $\dfrac{y^2}{4}$ C $\dfrac{y^2}{8}$ D $\dfrac{y^2}{16}$ E $\dfrac{y^2}{32}$

19. The diagram shows two squares and four equal semicircles. The edges of the outer square have length 48 and the inner square joins the midpoints of the edges of the outer square. Each semicircle touches two edges of the outer square, and the diameter of each semicircle lies along an edge of the inner square.

What is the radius of each semicircle?

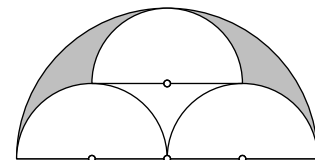   A 10   B 12   C 14   D 16   E 18
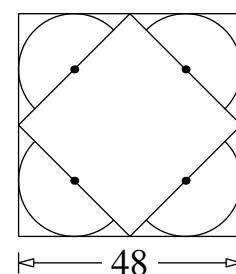
20. For any fixed value of x, which of the following four expressions has the largest value?

$$(x + 1)(x - 1) \qquad (x + \tfrac{1}{2})(x - \tfrac{1}{2}) \qquad (x + \tfrac{1}{3})(x - \tfrac{1}{3}) \qquad (x + \tfrac{1}{4})(x - \tfrac{1}{4})$$

A $(x + 1)(x - 1)$ B $(x + \tfrac{1}{2})(x - \tfrac{1}{2})$ C $(x + \tfrac{1}{3})(x - \tfrac{1}{3})$ D $(x + \tfrac{1}{4})(x - \tfrac{1}{4})$

E it depends on the value of x

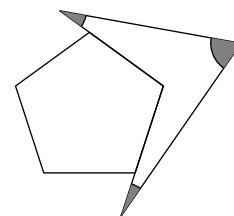**21.** The diagram shows four semicircles, one with radius 2 cm, touching the other three, which have radius 1 cm.

What is the total area, in cm^2, of the shaded regions?

A 1   B $\pi - 2$ C $2\pi - 5$ D $\tfrac{3}{2}$ E $\tfrac{1}{2}\pi$
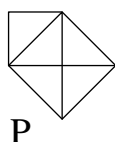
22. The diagram shows a regular pentagon and an irregular quadrilateral.

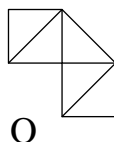What is the sum of the three marked angles?
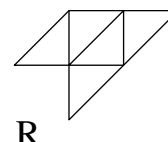
A 72°   B 90°   C 108°   D 126°   E 144°

23. Five congruent triangles, each of which is half a square, are placed together edge to edge in three different ways as shown to form shapes P, Q and R.

P   Q   R

Which of the following lists gives the shapes in ascending order of the lengths of their perimeters?

A P, Q, R   B Q, P, R   C R, Q, P   D R, P, Q   E P, R, Q

24. The positive integers m and n are such that $10 \times 2^m = 2^n + 2^{n+2}$.
What is the difference between m and n?

A 1   B 2   C 3   D 4   E 5

25. The diagram shows six points P, Q, R, S, T and U equally spaced around a circle of radius 2 cm. The inner circle has radius 1 cm. The shaded region has three lines of symmetry.
Which of the following gives the area, in cm^2, of the shaded region?

A $2\pi + 3$ B $3\pi + 2$ C $\dfrac{4\pi + 3}{2}$ D $3(\pi + 2)$ E $4\pi + 3$